

MATH 1650: SECTION 1.4: QUADRATIC FUNCTIONS

QUADRATIC FUNCTION: There are two common ways to describe quadratic functions:

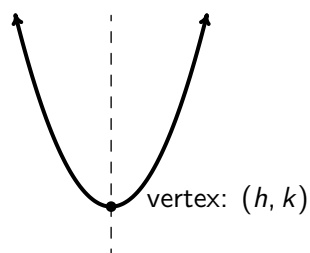
GENERAL FORM: The **general form** of a quadratic function is $f(x) = ax^2 + bx + c$, $a \neq 0$.

STANDARD FORM: The **standard form** of a quadratic function is $f(x) = a(x - h)^2 + k$, $a \neq 0$.

NOTE: To convert from general form to standard form, you need to **complete the square**.

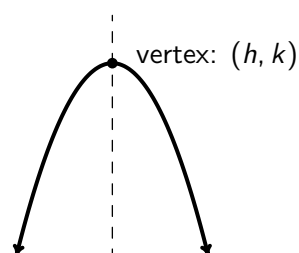
FACTS ABOUT QUADRATIC FUNCTIONS:

Graphs of quadratic functions are **parabolas**.



Axis of Symmetry: $x = h$

$a > 0$; Minimum: k , no maximum



Axis of Symmetry: $x = h$

$a < 0$; Maximum: k , no minimum

FORMULAS FOR VERTEX AND ZEROS:

Standard Form:

$$f(x) = a(x - h)^2 + k$$

- Vertex: (h, k)
- To solve $f(x) = 0$:
 - isolate $(x - h)^2$
 - extract square roots.

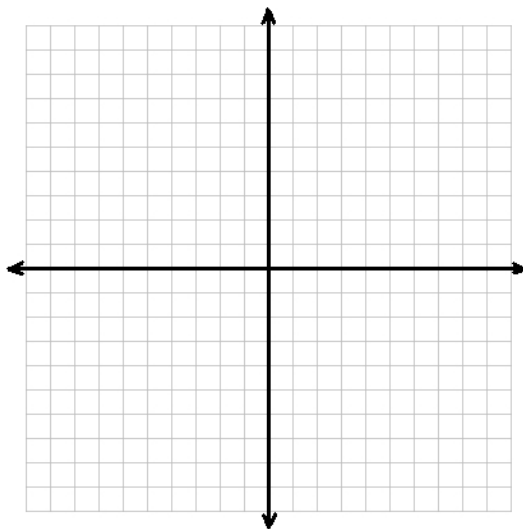
General Form:

$$f(x) = ax^2 + bx + c$$

- Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
- To solve $f(x) = 0$:
 - try factoring or else:
 - use: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXAMPLE: Let $f(x) = 2(x - 1)^2 + 3$.

- Find the vertex of the graph of $y = f(x)$.
- Find $f(0)$ and use this to find the y -intercept of the graph of $y = f(x)$.
- Show $f(x) = 0$ has no real solutions and explain why this means the graph of $y = f(x)$ has no x -intercepts.
- Graph $y = f(x)$ using the vertex, y -intercept, and symmetry.



the **domain** of f :

the **range** of f :

the **maximum** of f :

the **minimum** of f :

f is:

increasing:

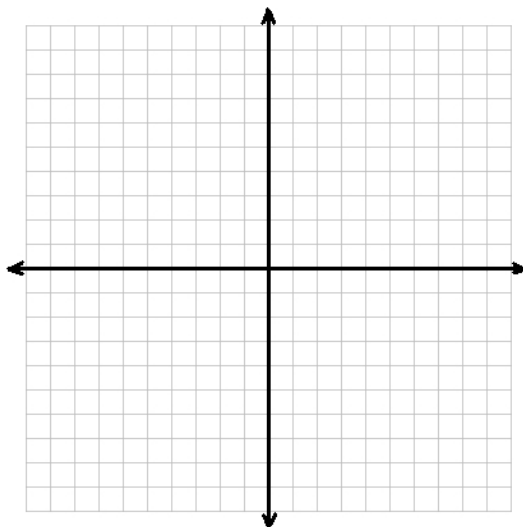
decreasing:

constant:

EXAMPLE: Let $f(x) = -\frac{1}{2}(x - 1)^2 + 8$.

- Find the vertex of the graph of $y = f(x)$.
- Find $f(0)$ and use this to find the y -intercept of the graph of $y = f(x)$.
- Solve $f(x) = 0$ and use these solutions to find the x -intercepts of the graph of $y = f(x)$.

- Graph $y = f(x)$ using the vertex, y -intercept, x -intercepts, and symmetry.



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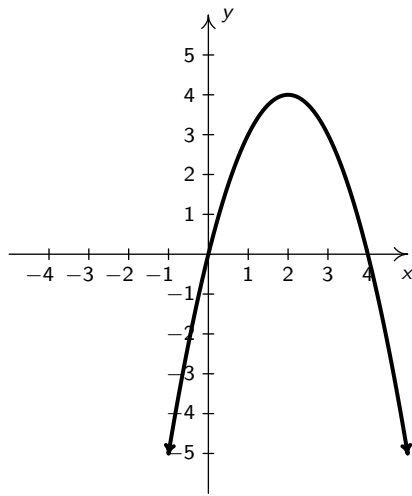
increasing:

decreasing:

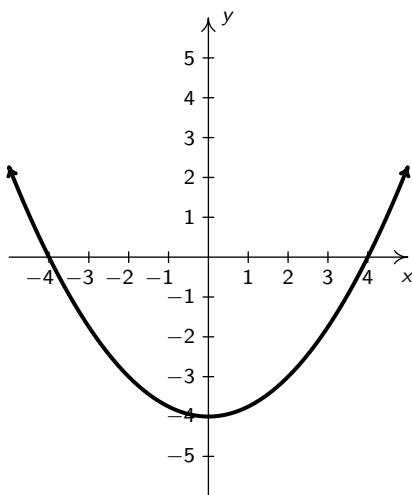
constant:

EXAMPLE: Find the **standard form** of the quadratic functions whose graphs are shown below:

- The graph of $y = f(x)$:

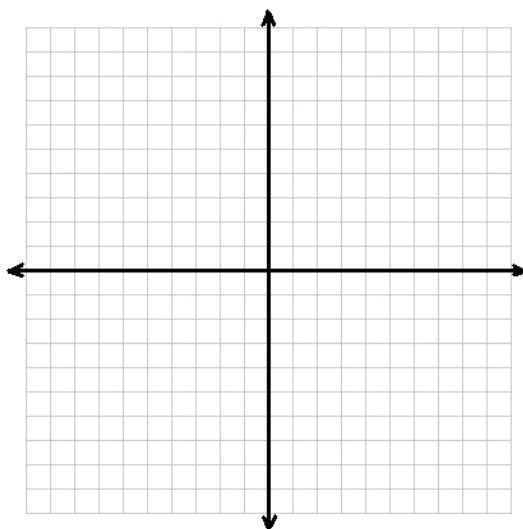


- The graph of $y = g(x)$:



EXAMPLE: Let $f(x) = 6x - x^2$.

- Use the vertex formula to find the vertex of the graph of $y = f(x)$.
- Find $f(0)$ and use this to find the y -intercept of the graph of $y = f(x)$.
- Solve $f(x) = 0$ and use these solutions to find the x -intercepts of the graph of $y = f(x)$.
- Graph $y = f(x)$ using the vertex, y -intercept, x -intercepts, and symmetry.



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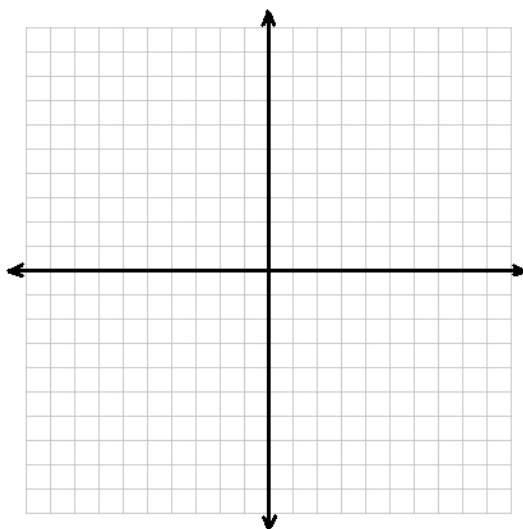
increasing:

decreasing:

constant:

EXAMPLE: Let $f(x) = x^2 - 3x - 4$.

- Use the vertex formula to find the vertex of the graph of $y = f(x)$.
- Find $f(0)$ and use this to find the y -intercept of the graph of $y = f(x)$.
- Solve $f(x) = 0$ and use these solutions to find the x -intercepts of the graph of $y = f(x)$.
- Graph $y = f(x)$ using the vertex, y -intercept, x -intercepts, and symmetry.



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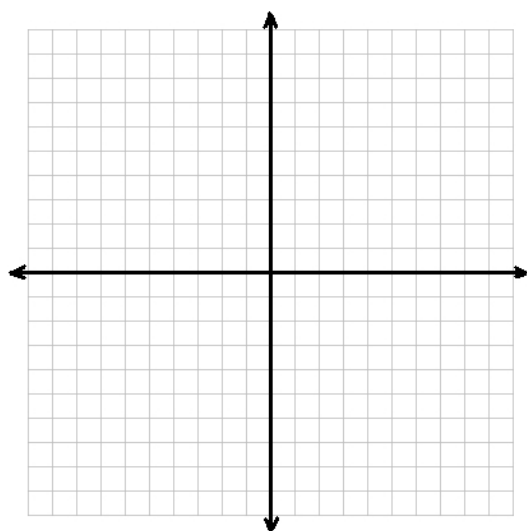
increasing:

decreasing:

constant:

EXAMPLE: Let $f(x) = 6 - 2x - x^2$.

- Use the vertex formula to find the vertex of the graph of $y = f(x)$.
- Find $f(0)$ and use this to find the y -intercept of the graph of $y = f(x)$.
- Solve $f(x) = 0$ and use these solutions to find the x -intercepts of the graph of $y = f(x)$.
- Graph $y = f(x)$ using the vertex, y -intercept, x -intercepts, and symmetry.



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MATH 1650: APPLICATIONS OF QUADRATIC FUNCTIONS

RECALL:

- The cost C (in dollars) to make x "I'd rather be a Sasquatch" T-shirts is: $C(x) = 2x + 26$, $x \geq 0$.
- The price charged per T-Shirt (in dollars per shirt) is $p(x) = 30 - 2x$, $0 \leq x \leq 15$.

DEFINITIONS:

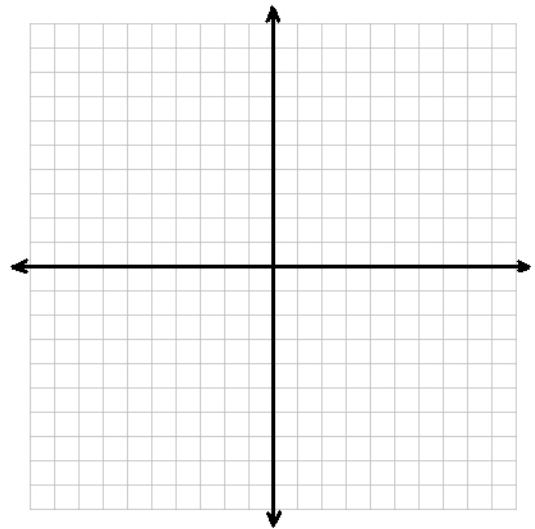
- **Revenue** is defined as: $\text{revenue} = (\text{number of items sold})(\text{price per item})$.
- **Profit** is defined as: $\text{profit} = \text{revenue} - \text{cost}$.

EXAMPLE: Assume we make exactly the same amount of T-shirts we sell, ' x .'

- Find an expression for $R(x)$, the revenue generated as a result of making and selling ' x ' T-shirts.
- Find an expression for $P(x)$, the profit earned as a result of making and selling ' x ' T-shirts.
- Find $P(0)$ and interpret what $P(0)$ means in terms of profits and shirts.
- Solve $P(x) = 0$ and interpret what the solutions mean in terms of profits and shirts.

- Use the vertex formula to find the vertex of the graph of $y = P(x)$. Interpret what each coordinate of the vertex means in terms of profits and shirts.

- Sketch the graph of $y = P(x)$. Label the intercepts and vertex.



- Find and interpret the average rate of change of P over the interval $[7, 14]$.